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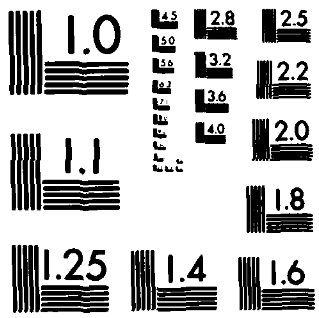

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This interim report constitutes a summary of research performed under Grant AFOSR-81-0047 during the year beginning 1 October 1981. First the author presents a list of the personnel involved in the research effort. In the next section he presents a summary of the research results that have been achieved. Then in the following section he briefly comments upon the research in progress. This is followed by a list of publications supported during this grant period.

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INTERIM REPORT

Grant AFOSR-81-0047

November 29, 1982

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Introduction

This interim report constitutes a summary of research performed under Grant AFOSR-81-0047 during the year beginning October 1, 1981. First we present a list of the personnel involved in the research effort. In the next section we present a summary of the research results that have been achieved. Then in the following section we briefly comment upon the research in progress. This is followed by a list of publications supported during this grant period.

Personnel

Principal Investigator:
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* Efren F. Abaya received his Ph.D. degree in August 1982. His dissertation was entitled "A Theoretical Study of Optimal Vector Quantizers."

Summary of Results

In this section we will present a brief summary of our research results published after October 1, 1981. In the following section we will give a brief overview of our research in progress. Then in the final section we will present a list of publications supported by the Grant AFOSR-81-0047 since October 1, 1981. The references in the present section are keyed to the publication list in the last section.

Much of our work during the previous grant year was concerned with quantization theory. Quantization forms the heart of analog to digital conversion and it is a key element in virtually all of digital signal processing. An N-level k-dimensional vector quantizer is a mapping $Q: \mathbb{R}^k \to \mathbb{R}^k$ which assigns the input vector x to an output vector Q(x) chosen from a finite set of N distinct vectors $\{y_i : y_i \in \mathbb{R}^k, i=1,2,\ldots,N\}$. Generally, the quantizer input is modelled as a random vector X described by a k-variate distribution F. A measure of quantizer performance is the distortion function

$$D(Q,F) = \int C[x - Q(x)]dF(x),$$

where $C(\cdot)$ is an appropriately chosen cost function, for example, $C(x) = \|x\|^2$, where $\|\cdot\|$ denotes the Euclidean norm on $\mathbb{R}^k$, results in the mean squared error. Thus the quantizing operation entails a coarsening or reduction of available information, in the sense that all points in the set $R_i = \{x : Q(x) = y_i\}$ are identified together.

Although quantization theory has been a subject of interest for many years, much remains to be done in this area. The difficulty in this area is compounded by the fact that some of the commonly held beliefs are actually misconceptions. For example, it has often been assumed in the literature that in the scalar case a minimum mean squared error *optimum* quantizer for a symmetrically distributed random variable will be symmetric (i.e. the quantization levels will be symmetrically distributed about the origin). In [10] we gave the following result for scalar quantizers.

<u>Theorem</u>: Let the cost function be given by $C(x) = |x|^r$ for some positive r. Let N be a positive even integer and let f_0 be a given symmetric density function with finite r-th absolute moment. For any given ε, $0 < \varepsilon < 1$, there

exists a symmetric density function h such that for the mixture density $(1-\varepsilon)f_0 + \varepsilon h$, no optimal N-level scalar quantizer is symmetric.

An open problem in the subject of quantization theory had been the existence of optimal quantizers (i.e. could the infimum over all N-level quantizers be achieved?). In [15] we showed that if $C(x) = C_0(||x||)$, where $C_0(t)$ is nonnegative, real valued, nondecreasing for $t \geq 0$, and lower semi-continuous, then an optimum N-level quantizer exists for any positive integer N. In [10] we extended this result to the case where if $C_0(t)$ is non-negative and non-decreasing for $t \geq 0$, then a necessary and sufficient condition that, for any N and any distribution function, a minimum distortion N-level quantizer exists, is that $C_0(t)$ be lower semi-continuous. Also in [2] we showed that if the distribution of the random vector X is continuous, then for any cost function $C(x) = C_0(||x||)$, where $C_0(t)$ is nonnegative and nondecreasing for $t \geq 0$, a globally minimum distortion N-level quantizer exists for every N. For example, the well-known Cantor distribution is continuous (although singular with respect to Lebesgue measure); nevertheless, any such cost function yields a distortion that can be minimized.

In [1] and [17] we investigated some convergence properties of sequences of quantizers (and as a side result we established existence of optimal quantizers via convergence arguments). Specifically, we were concerned with the following. Suppose that a sequence $\{F_n\}$ of probability distribution functions on $\mathbb{R}^k$ converges weakly to a distribution function F. When does the sequence of optimal quantizers $\{Q_n\}$ for the F_n's converge to an optimal quantizer Q for F? When does the sequence of distortions $D(Q_n,F_n)$ converge to $D(Q,F)$? In [1] and [17] we discussed conditions under which this convergence can take place. In recent years much work has been done to investigate design algorithms for optimal quantization. A popular approach is to use numerical techniques to successively improve on a sequence of sub-optimal designs using conditions which are necessary (but not sufficient) for a stationary point. Linde, Buzo, and Gray ("An Algorithm for Vector Quantizer Design," IEEE Transactions on Communications, Vol. COM-28, pp. 84-95, January 1980) recently presented an algorithm (essentially a fixed point algorithm) for quantizer design that handles arbitrary multi-dimensional distributions and very general distortion measures, and they conjectured that this algorithm would converge to a global minimum but were unable to prove it rigorously. In [1] and [17] we proved convergence of this algorithm for quantizer design.

In [6] we considered asymptotic properties of the r-th power distortion,

i.e. $E\{\| X-Q(X)\|^r\}$, associated with quantized k-dimensional random variables. Subject only to a moment condition,we showed in [6] that the infimum over all N-level quantizers of the quantity $N^{r/k}$ times the r-th power distortion measure converges to a finite constant as $N \to \infty$. This result might be utilized in the following way. Let Q_N denote an optimal N-level quantizer. How big should N be so that the r-th power distortion is acceptably small? Since $N^{r/k} E\{\| X-Q_N(X)\|^r\} \to k$, if N is sufficiently large we might invoke the approximation $E\{\| X-Q_N(X)\|^r\} \approx k N^{-r/k}$. Notice that our work in [6], where we considered an example involving the Cantor distribution, stands in marked contrast to the following statement, taken from Yamada, Tazaki, and Gray ("Asymptotic Performance of Block Quantizers with Difference Distortion Measures," IEEE Transactions on Information Theory, Vol. IT-26, pp. 6-14, January 1980): "The initial fundamental assumption in all studies of asymptotic quantization is that the probability density p(x) is sufficiently 'smooth' to ensure that p(x) is effectively constant over small bounded sets."

In the context of scalar quantizers, any quantizer can be realized via the companding approach; that is, a strictly increasing nonlinearity (the compressor) mapping the reals into [0,1] followed by a uniform quantizer on [0,1] followed in turn by the inverse of the first nonlinearity (the expander). In [16] we investigated the situation where the output of the uniform quantizer is transmitted through a noisy channel and the output of this channel is then put through the expander. The compressor was designed taking into account both the effects of quantization errors and channel errors. Then in [3] we considered a slightly different approach to companding, where the compander was constrained to be piecewise linear. We designed the (asymptotically) optimal compander subject to the constraint that it be piecewise linear, and we investigated several properties of such piecewise linear companders.

In [11] we investigated an existing method (E.J. Delp and O.R. Mitchell, "Image Compression Using Block Truncation Coding," IEEE Transactions on Communications, Vol. COM-27, pp. 1335-1342, September 1979) for image compression known as block truncation coding. The basic block truncation coding approach employs a one bit (i.e. two level) nonparametric quantizer whose output levels are obtained through matching the first two sample moments of the data before and after quantization. In [11] we generalized this basic block truncation coding approach by employing one bit nonparametric quantizers which preserve higher order moments. This generalization offered the potential for improved performance. We illustrated by way of example that such improvement was

indeed possible under the criteria of mean absolute error and/or mean squared error.

The matched filter has been of practical interest for some time. This filter, which maximizes the output signal to noise power ratio, requires knowledge of the signal input for its design. In many cases it is reasonable to expect that the signal will be known at various discrete instants, thus admitting the design of the discrete time filter. Unfortunately, while it is reasonable to expect that the input signal will be known at discrete instants, it is another matter to assume that it will be known exactly as a closed form analytical expression over, for example, an interval of time, which would be necessary for the design of a matched filter in continuous time. Design of the continuous time filter is thus inhibited by such inexact knowledge of the signal input.

While the signal may be incompletely known, it is reasonable to expect that in many cases it could be modeled as bandlimited. If we furthermore assume the signal is known at a fixed number of instants, we might hope that a continuous time filter could be designed which is insensitive to the remaining inexactness in our knowledge of the signal. In [2] and [18] we presented an approach toward inproving the signal to noise power ratio of a discrete time matched filter by employing a continuous time filter. This approach exploited the bandlimited nature of the unknown continuous time signal, and resulted in a filter which is robust to this inexact knowledge.

The relative efficiency between two detectors is a measure of the amount of data one detector requires, relative to the other detector, to attain a prescribed level of performance. Often the asymptotic relative efficiency (ARE) is employed as a criterion for comparison of detectors. (In the context of hypothesis testing, this is known in the statistical literature as the Pitman efficiency.) The ARE is generally held to be appropriate in the case of large sample size and small signal strength. Moreover, the employment of the ARE generally yields mathematically tractable results, due largely to the applicability of central limit theorems. While it is not our intention to disqualify the ARE as a measure of detector performance, our investigations do question its validity as a universal measure of detector performance. In [9] we investigated the behavior of two pairs of popular detection systems, and we found that in some cases the ARE can be a poor indicator of finite sample size detection performance even for some fairly large sample sizes. Thus, in some cases if the relative efficiency is assumed to converge quickly, the "wrong" detector might

be chosen for a particular application. This behavior was seen to be particularly true for heavy-tailed noise models such as Laplace noise.

Kassam and Thomas ("A Class of Nonparametric Detectors for Dependent Input Data," IEEE Transactions on Information Theory, Vol. IT-21, pp. 431-437, July 1975) considered detecting a constant signal in m-dependent noise. This scheme consisted of summing the first n samples, skipping (i.e. throwing away) the next m, summing the next n, skipping the next m, etc. They then applied the classical sign detector to the sequence of sums, and they concluded that, asymptotically, n should be chosen as large as possible to maximize performance. In [8] we investigated the performance of this nonparametric scheme in both the asymptotic case and the case of a finite number of samples. We showed that the design of the detector with a finite amount of data can be radically different from the design based on the asymptotic limit.

In several applications of discrete time signal processing, a nonlinear scheme called median filtering has achieved some very interesting results. The implementation of a median filter requires a very simple nonlinear operation. Consider a fixed nonnegative integer N. At a given time instant, the output of the median filter is the empirical median of the samples lying within a window centered at the given time instant and spanning 2N+1 adjacent samples. Tukey is generally credited with introducing the concept of median filters, and he did much of the pioneering work in this area. Although the concept of median filters was first introduced in the statistics literature, it soon found applications in the area of signal processing. Loosely speaking, linear filtering tends to smooth out abrupt changes in a sequence of data, but a median filter is capable of preserving sharp changes in data. In [4] we presented the first rigorous analysis of the properties of median filters. We precisely characterized which signals could pass through a median filter unchanged, we characterized the concept of a root of a median filter, and we showed that with repeated median filtering we would eventually produce a root signal of the median filter. In [5] we considered the effect of a median filter upon the spectral density of a homogeneous Markov chain. In [12] we analyzed the effect of a median filter on an input which consisted of the sum of a known constant signal and independent identically distributed noise. Specifically, we studied the probability that the difference between the output of the filter and the constant signal is close to zero. This probability can be used to determine the performance of the filter on a per-sample basis. Some examples were presented to illustrate the results.

A model that frequently arises in engineering applications is given by the differential equation

$$\dot{X}(t) = A(t)X(t)$$

where $t \geq 0$, $X(t) \in \mathbb{R}^k$, $X(0) = x_0$, and $A(t)$ is a $k \times k$ random matrix. For example, by taking

$$A(t) = G + N(t)H,$$

where $N(t)$ is a random process and H and G are $k \times k$ matrices, we arrive at a bilinear system. (Bilinear systems and their relations to more general nonlinear systems, e.g. Volterra systems, have been investigated by other researchers.) Systems of this general form also arise in modelling faulty systems (the random changing of $A(t)$ is used to represent the effects of faulty devices in a system). Numerous investigations have been concerned with the establishment of conditions guaranteeing that as $t \to \infty$, $\| X(t)\| \to 0$ in some probabilistic sense, where $\|\cdot\|$ is some norm on $\mathbb{R}^k$. We have recently established conditions [13] guaranteeing that $\| X(t)\| \to 0$ with probability one, and these conditions are more general than others dealing with this form of problem. We do not require the components of $A(t)$ to be ergodic, and we permit them to even possess "badly behaved" sample functions; for example, sample functions possessing a discontinuity of the second kind at every point and that are unbounded in any interval of positive length. Our conditions are expressed in terms of second moment properties of the components of $A(t)$.

In [7] we considered the prediction of a time series, where our predictor was constrained to have the form of a zero memory nonlinearity followed by a linear filter. Systems such as this can always perform at least as well as linear systems, and in many cases significantly outperform linear systems.

Minimum shift keying (MSK) is a popular digital modulation technique which is known for its property of bandwidth conservation. Boutin and Morissette ("Do All MSK-type Signalling Waveforms Have Wider Spectra than those for PSK?," IEEE Transactions on Communications, Vol. COM-29, pp. 1071-1072, July 1981) made an erroneous attempt to provide a bound on the width of the center spectral lobe of MSK-type signaling waveforms. In [14] we provided a correct development of this property which yielded a bound on the width of the center spectral lobe of MSK-type signaling waveforms.

Research in Progress

During the present grant year our research is concentrating upon several problem areas. One problem area which we are vigorously pursuing is quantization theory. Our earlier work had dealt with difference-based distortion measures, that is, distortions such as

$$\int C[x-Q(x)]dF(x).$$

We are presently extending this to nondifference-based distortion measures of the form

$$\int d[x,Q(x)]dF(x).$$

It is known that difference-based distortions are not always satisfactory, especially when held up against subjective evaluations. Recently, more complicated cost functions have been investigated in an effort to obtain more meaningful distortion measures. We plan to investigate existence of optimal quantizers and also convergence properties of sequences of quantizers. Also, we plan to extend the space being quantized from $\mathbb{R}^k$ to metric spaces. Quantization of metric spaces may be potentially useful as a tool in extending theorems in information theory and coding theory to function spaces.

Another problem area we will be concerned with is the theory of signal detection. We plan to study several questions concerned with approximations in detection systems; for example, the digital implementation of an analog scheme for detection. Also, we plan to study the relative efficiency (i.e. with a finite number of samples) between pairs of detectors for time-varying signals.

Another problem we are considering is concerned with the effect of a zero memory nonlinearity on the spectrum of certain classes of random processes. That is, let $X(t)$ be a stationary random process possessing a spectral representation, and let $g: \mathbb{R} \to \mathbb{R}$ be a Borel measurable function such that $E\{(g[X(t)])^2\} < \infty$. Then what can we say about the spectrum of $g[X(t)]$ as compared to that of $X(t)$? When is one more spread out than the other? This problem thus relates to the bandwidth of nonlinearly distorted random processes.

List of Publications

1. E.F. Abaya and G.L. Wise, "Convergence of Vector Quantizers with Application to the Design of Optimal Quantizers," Proceedings of the Nineteenth Annual Allerton Conference on Communication, Control, and Computing, Monticello, Illinois, September 30-October 2, 1981, pp. 79-88.

2. D.R. Halverson and G.L. Wise, "An Observation Concerning Signal to Noise Ratio Properties of Continous and Discrete Time Detectors," Proceedings of the Nineteenth Annual Allerton Conference on Communication, Control, and Computing, Monticello, Illinois, September 30-October 2, 1981, pp. 569-575.

3. F. Kuhlmann and G.L. Wise, "Design Considerations for Asymptotically Optimal Piecewise Linear Companders, "Proceedings of the 1981 National Telecommunications Conference, New Orleans, Louisiana, November 29-December 3, 1981, pp. F4.4.1-F4.4.4.

4. N.C. Gallagher and G.L. Wise, "A Theoretical Analysis of the Properties of Median Filters," IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-29, pp. 1136-1141, December 1981.

5. F. Kuhlmann and G.L. Wise, "The Bivariate Distribution of a Median Smoothed Markov Chain," Journal of the Franklin Institute, Vol. 313, pp. 107-118, February 1982.

6. J.A. Bucklew and G.L. Wise, "Multidimensional Asymptotic Quantization Theory with r-th Power Distortion Measures," IEEE Transactions on Information Theory, Vol. IT-28, pp. 239-247, March 1982.

7. T.E. McCannon, N.C. Gallagher, D. Minoo-Hamedani, and G.L. Wise, "On the Design of Nonlinear Discrete Time Predictors," IEEE Transactions on Information Theory, Vol. IT-28, pp. 366-371, March 1982.

8. D.R. Halverson and G.L. Wise, "On the Performance of a Nonparametric Detection Scheme for Dependent Data," IEEE Transactions on Information Theory, Vol. IT-28, pp. 380-384, March 1982.

9. D.L. Michalsky, G.L. Wise, and H.V. Poor, "A Relative Efficiency Study of Some Popular Detectors," Journal of the Franklin Institute, Vol. 313, pp. 135-148, March 1982.

10. E.F. Abaya and G.L. Wise, "Some Remarks on Optimal Quantization," Proceedings of the 1982 Conference on Information Sciences and Systems, Princeton, New Jersey, March 17-19, 1982., pp. 60-65.

11. D.R. Halverson, N.C. Griswold, and G.L. Wise, "On Generalized Block Truncation Coding Quantizers for Image Compression," Proceedings of the 1982 Conference on Information Sciences and Systems, Princeton, New Jersey, March 17-19, 1982, pp. 47-52.

12. F. Kuhlmann and G.L. Wise, "Performance of Median Filters with Random Inputs," Proceedings of the 1982 International Conference on Communications, Philadelphia, Pennsylvania, June 13-17, 1982, pp. IH.2.1-IH.2.5.

13. G.L. Wise, "A Result on the Stability of Linear Differential Equations with Random Coefficients," Systems and Control Letters, Vol. 1, pp. 385-389, May 1982.

14. D.R. Halverson and G.L. Wise, "A Further Comment on the Spectral Width of MSK-type Signaling Waveforms," IEEE Transactions on Communications, Vol. COM-30, pp. 1983-1984, August, 1982.

15. E.F. Abaya and G.L. Wise, "On the Existence of Optimal Quantizers," IEEE Transactions on Information Theory, Vol. IT-28, pp. 937-940, November 1982.

16. F. Kuhlmann, J.A. Bucklew, and G.L. Wise, "Compressors for Combined Source and Channel Coding with Applications to the Generalized Gaussian Family," to appear in IEEE Transactions on Information Theory, January 1983.

17. E.F. Abaya and G.L. Wise, "Convergence of Vector Quantizers with Applications to Optimal Quantization," to appear in SIAM Journal on Applied Mathematics.

18. D.R. Halverson and G.L. Wise, "On an Aspect of Matched Filters in Continuous and Discrete Time," to appear in Circuits, Systems, and Signal Processing.


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